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Validation of Analytical Methods for Estimating Critical Speeds in Shafts with Concentrated Masses Using Computational Simulation

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ABSTRACT

Rayleigh's method, Dunkerley's equation, Euler-Bernoulli solutions, and finite element simulation in SolidWorks are compared to estimate the first flexural critical speed of uniform-section AISI 4140 steel shafts. Three configurations are studied: an overhung shaft with a mass at its free end, a simply supported shaft with a central mass, and a simply supported shaft with two symmetric masses. Analytical calculations are presented and their results are compared with the frequencies obtained by simulation. Dunkerley's estimates are close to the numerical reference values; in the two-mass case, Rayleigh's method shows a larger difference. The comparison allows the assumptions of the models and the discrepancies between their predictions to be examined. Potential applications to preliminary design and teaching activities that connect analytical calculation with simulation are also discussed.

KEYWORDS

Finite element analysis; modal analysis; numerical simulation; shafts; vibrations; engineering education

INTRODUCTION

The precise determination of critical speeds is a fundamental aspect in the design of transmission shafts for rotating machinery. Operation close to these natural flexural frequencies can cause resonance, excessive deflections, and catastrophic equipment failure. Traditional analytical methods, such as Rayleigh's method and Dunkerley's equation, offer quick and conservative estimates of the first flexural critical speed. However, these techniques rely on simplifications (static deflection curves, concentrated masses, ideal support conditions) that may overestimate or underestimate the actual frequency [1], [2], [3].

Finite element simulation allows calculating natural frequencies and visualizing vibration modes. Its comparison with analytical methods facilitates the examination of the model's hypotheses and the identification of discrepancies. In education, both procedures can be used in a complementary manner to connect the fundamentals of calculus with the numerical representation of the system [1], [2], [4].

The estimation of critical speeds is part of the analysis of shafts used in rotating machinery [5], [6]. This article studies three configurations with one or several concentrated masses. The influence coefficients, individual critical speeds, and Rayleigh and Dunkerley calculations corresponding to each case are presented and compared with SolidWorks results. The discussion considers the differences between both procedures and their potential application to preliminary design and education.

SCOPE OF THE WORK

The study compares the analytical calculation of flexural critical speeds with finite element simulation in SolidWorks. The models use the shaft geometry, the modulus of elasticity and density of AISI 4140 steel, the section's moment of inertia, and the concentrated masses. From these parameters, frequencies are calculated, and the results of each configuration are contrasted.

GENERAL AND SPECIFIC LEARNING OBJECTIVES

General objective:

To compare the estimates of the first flexural critical speed obtained through Rayleigh and Dunkerley methods with finite element simulation results, and to discuss their use in preliminary design and mechanical engineering education.

Specific objectives:

Cognitive Domain:

Calculate the first natural flexural frequency of three shaft configurations with concentrated masses using the analytical expressions developed for each case.

Compare the analytical results with those obtained through finite element simulation in SolidWorks and calculate their relative differences.

Examine the differences between the Rayleigh, Dunkerley, and finite element simulation results, considering the hypotheses of each model.

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Psychomotor Domain:

Build the geometric models of the shaft in SolidWorks and assign the support conditions and remote masses for each configuration.

Configure and execute a frequency study in SolidWorks, defining the mesh, selecting the solver, and interpreting the vibration.

Affective Domain:

Value the comparison between analytical calculation and simulation as a resource to review the models used in the design of mechanical elements.

Promote the joint use of analytical calculation and simulation tools in the training of mechanical and mechatronic engineers.

OBJECT OF STUDY

The object of study is the first natural flexural frequency of a uniform circular cross-section shaft made of AISI 4140 steel, used to estimate its first critical speed. Three configurations are considered: an overhung shaft with a mass at the free end, a simply supported shaft with a central mass, and a simply supported shaft with two symmetric masses relative to the center. Calculations based on influence coefficients, Rayleigh, and Dunkerley are compared with finite element simulation.

METHODOLOGY

A comparative study was conducted using analytical calculation and finite element simulation of three shaft configurations with concentrated masses. The geometry, material properties, and support conditions were defined; then the frequencies were calculated and the results from both procedures were compared. The simulation was used as a numerical reference to estimate the relative differences.

Description of the Case Studies

An AISI 4140 steel shaft with the parameters from Table 1 is considered.

Table 1.
Parameters and characteristics of the prototype elements

CONCEPT	VARIABLE	VALUE
Diameter	d	$d = 15.875\,\text{mm}$
Distance between supports	L	$L = 0.6\,\text{m}$
Modulus of Elasticity	E	$E = 2.05 \times 10^{11}\,\text{Pa}$
Density	ρ	$\rho = 7850\,\text{kg/m}^3$
Moment of Inertia	I	$I = 3.118 \times 10^{-9}\,\text{m}^4$
Flywheel Mass	m	$m = 2.5\,\text{kg}$

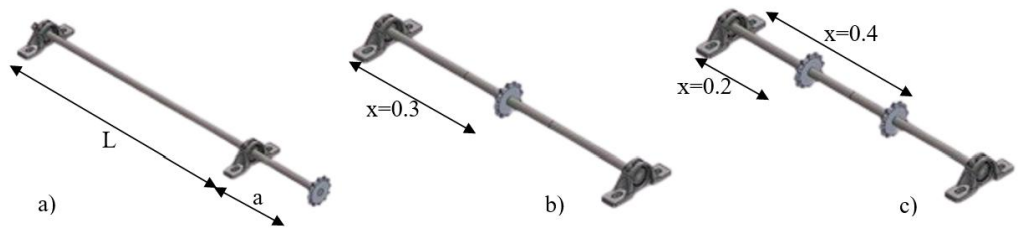
Three specific configurations are analyzed, as shown in Figure 1.

Simply supported shaft with overhang: overhang length $a = L/3 = 0.2\,\text{m}$, mass at the free.

Simply supported shaft: a mass in the center of the span ($x = 0.3\,\text{m}$).

Simply supported shaft: two equal masses located at $x = 0.2\,\text{m}$ and $x = 0.4\,\text{m}$.

Figure 1.
Analyzed configurations: a) overhung shaft; b) simply supported shaft with a central mass; c) simply supported shaft with two central masses



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DEVELOPMENT PHASES

Phase 1. Observation and problem statement:

Identification of the resonance problem in transmission shafts and review of available classical analytical methods (Rayleigh, Dunkerley, Euler-Bernoulli).

Phase 2. Definition of the object of study:

Definition of the three shaft configurations with concentrated masses and the material and geometry properties used in the calculations (Table 1).

Phase 3. Analytical calculation:

Step-by-step development of influence coefficients, static deflections, and individual frequencies for each configuration, combined using Rayleigh and Dunkerley equations.

Phase 4. Computational simulation:

CAD modeling of each configuration in SolidWorks, material and boundary condition assignment, application of remote masses, and execution of a Frequency Study.

Phase 5. Comparative analysis:

Comparison of analytical results with those from finite element simulation, calculation of relative percentage differences, and examination of hypotheses that might explain the discrepancies.

Phase 6. Conclusion and implications:

Synthesis of results and discussion of their possible applications in education and preliminary design.

Detailed Analytical Development for Each Case

The calculations for each configuration are presented, starting with general properties.

General properties:

$$\begin{aligned} A &= \frac{\pi d^2}{4} = 1.979 \times 10^{-4}\ \mathrm{m}^2, \\ EI &= 2.05 \times 10^{11} \times 3.118 \times 10^{-9} = 639.2\ \mathrm{N\cdot m}^2, \\ k &= \rho A g = 7850 \times 1.979 \times 10^{-4} \times 9.81 = 15.24\ \mathrm{N/m}, \\ g &= 9.81\ \mathrm{m/s}^2. \end{aligned} \tag{1}$$

Case 1: Overhung shaft with mass at the end

For the massless shaft, the frequency obtained by FEM is taken as the numerical reference:

$$\omega_s^{\mathrm{FEM}} = 527.2\ \mathrm{rad/s}, \quad f_s = 83.903\ \mathrm{Hz}, \quad n_s = 5034\ \mathrm{rpm}. \tag{2}$$

ω_s^{FEM} is not calculated directly with a simple formula, but by solving the eigenvalue problem of the system:

$$[K]\{\phi\} = \omega^2[M]\{\phi\} \tag{3}$$

where:

[K]= global stiffness matrix

[M] = global mass matrix

{ϕ} = modal vector

ω = angular natural frequency (rad/s)

After discretizing the shaft into beam elements:

$$[M] = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \quad ; \quad [K] = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \tag{4}$$

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The characteristic problem is formulated:

$$\det\left(\left[K\right]-\omega^2\left[M\right]\right)=0 \tag{5}$$

By expanding the determinant, a polynomial in ω² is obtained:

$$a\omega^4+b\omega^2+c=0 \tag{6}$$

Assuming the solution of the eigenvalue problem is:

$$\omega_1^2=277939 \tag{7}$$

Then:

$$\begin{aligned}\omega_1&=\sqrt{277939}\\ \omega_1&=527.2\ rad/s\end{aligned} \tag{8}$$

Another approximation of the system is:

$$\begin{aligned}m&=2\ kg\\ k&=555878\ N/m\end{aligned} \tag{9}$$

Then:

$$\begin{aligned}\omega_n&=\sqrt{\frac{k}{m}}\\ \omega_n&=\sqrt{\frac{555878}{2}}=\sqrt{277939}\\ \omega_n&=527.2\ rad/s\end{aligned} \tag{10}$$

Disk only: Influence coefficient at the free end:

$$\delta_{CC}=\frac{a^2(L+a)}{3EI}=\frac{0.2^2\times 0.8}{3\times 639.2}=1.6688\times 10^{-5}\ \text{m/N}. \tag{11}$$

Deflection under the disk's weight:

$$y_d=W_d\,\delta_{CC}=24.525\times 1.6688\times 10^{-5}=4.092\times 10^{-4}\ \text{m}. \tag{12}$$

Individual frequency of the disk:

$$\omega_d=\sqrt{\frac{g}{y_d}}=\sqrt{\frac{9.81}{4.092\times 10^{-4}}}=154.8\ \text{rad/s},\quad f_s=24.637\ \text{Hz},\quad n_d=1478\ \text{rpm}. \tag{13}$$

Assembly (Dunkerley): Applying the modified Dunkerley equation:

$$\begin{aligned}\frac{1}{\omega_1^2}\approx\frac{1}{\omega_s^2}+\frac{1}{\omega_d^2}&=\frac{1}{527.2^2}+\frac{1}{154.8^2}\\ &=3.598\times 10^{-6}+4.172\times 10^{-5}=4.532\times 10^{-5},\end{aligned} \tag{14}$$

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$$\omega_1 = \frac{1}{\sqrt{4.532\times10^{-5}}} = 148.5\ \text{rad/s}, \quad f_1 = 23.64\ \text{Hz}, \quad n_1 = 1418\ \text{rpm}. \tag{15}$$

Case 2: Simply supported shaft with central mass.

Shaft only (exact): Using the equation:

$$\omega_s = \left(\frac{\pi}{L}\right)^2 \sqrt{\frac{EI}{\rho A}} \tag{16}$$

$$\omega_s = \left(\frac{\pi}{0.6}\right)^2 \sqrt{\frac{639.2}{1.553}} = 556.1\ \text{rad/s}, \quad f_s = 88.53\ \text{Hz}, \quad n_s = 5312\ \text{rpm}. \tag{17}$$

Disk only: Influence coefficient at the center [7]:

$$\delta_{cc} = \frac{L^3}{48EI} = \frac{0.6^3}{48 \times 639.2} = 7.040 \times 10^{-6}\ \text{m/N}. \tag{18}$$

Deflection:

$$y_d = W\delta_{cc} = 24.525 \times 7.040 \times 10^{-6} = 1.727 \times 10^{-4}\ \text{m}. \tag{19}$$

Individual frequency of the disk:

$$\omega_d = \sqrt{\frac{9.81}{1.727 \times 10^{-4}}} = 238.4\ \text{rad/s}, \quad f_1 = 37.9425\ Hz, \quad n_d = 2277\ \text{rpm}. \tag{20}$$

Assembly (Dunkerley):

$$\frac{1}{\omega_1^2} = \frac{1}{\omega_s^2} + \frac{1}{\omega_d^2} = \frac{1}{556.1^2} + \frac{1}{238.4^2} = 3.233 \times 10^{-6} + 1.760 \times 10^{-5} = 2.083 \times 10^{-5}, \tag{21}$$

$$\omega_1 = \frac{1}{\sqrt{2.083 \times 10^{-5}}} = 219.1\ \text{rad/s}, \quad f_1 = 34.87\ \text{Hz}, \quad n_1 = 2093\ \text{rpm}. \tag{22}$$

Case 3: Simply supported shaft with two symmetric centered masses.

Shaft only: The value obtained by FEM is taken for direct comparison:

$$\omega_s = 546.9\ \text{rad/s}, \quad f_s = 87.059\ \text{Hz}. \tag{23}$$

Influence coefficients: With $a_1 = 0.2\ \text{m}$, $b_1 = 0.4\ \text{m}$, $a_2 = 0.4\ \text{m}$, $b_2 = 0.2\ \text{m}$ and common denominator $6EI\,L = 2301.12$, equation (7-24) from [7] is used:

$$\delta_{11} = \frac{b_1 x_1}{6EI\,L}(L^2 - b_1^2 - x_1^2) = \frac{0.4 \times 0.2}{2301.12}(0.36 - 0.16 - 0.04) = 5.563 \times 10^{-6}\ \text{m/N}. \tag{24}$$

$$\delta_{22} = \delta_{11} = 5.563 \times 10^{-6}\ \text{m/N}. \tag{25}$$

$$\delta_{12} = \delta_{21} = \frac{b_2 x_1}{6EI\,L}(L^2 - b_2^2 - x_1^2) = \frac{0.2 \times 0.2}{2301.12}(0.36 - 0.04 - 0.04) = 4.867 \times 10^{-6}\ \text{m/N}. \tag{26}$$

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Individual masses:

$$y_{11} = W\delta_{11} = 24.525 \times 5.563 \times 10^{-6} = 1.364 \times 10^{-4}\ \text{m}, \tag{27}$$

$$\omega_{11} = \sqrt{\frac{g}{y_{11}}} = \sqrt{\frac{9.81}{1.364 \times 10^{-4}}} = 268.2\ \text{rad/s}\ (n = 2560\ \text{rpm}). \tag{28}$$

The second identical mass yields the same value.

Rayleigh's Method: The static deflections due to both weights are equal:

$$y_1 = y_2 = W(\delta_{11} + \delta_{12}) = 24.525 \times (1.043 \times 10^{-5}) = 2.558 \times 10^{-4}\ \text{m}. \tag{29}$$

Summations:

$$\begin{aligned} \sum W_i y_i &= 2 \times 24.525 \times 2.558 \times 10^{-4} = 1.255 \times 10^{-2}\ \text{N·m}, \\ \sum W_i y_i^2 &= 2 \times 24.525 \times (2.558 \times 10^{-4})^2 = 3.210 \times 10^{-6}\ \text{N·m}^2. \end{aligned} \tag{30}$$

Applying Rayleigh:

$$\omega_1^{\text{Ray}} = \sqrt{\frac{g\sum W_i y_i}{\sum W_i y_i^2}} = \sqrt{\frac{9.81 \times 1.255 \times 10^{-2}}{3.210 \times 10^{-6}}} = 195.8\ \text{rad/s}, \quad f = 31.17\ \text{Hz}, \quad n = 1870\ \text{rpm}. \tag{31}$$

Dunkerley:

$$\begin{aligned} \frac{1}{\omega_1^2} &= \frac{1}{\omega_s^2} + \frac{1}{\omega_{11}^2} + \frac{1}{\omega_{22}^2} = \frac{1}{546.9^2} + \frac{2}{268.2^2} \\ &= 3.344 \times 10^{-6} + 2.782 \times 10^{-5} = 3.116 \times 10^{-5}, \end{aligned} \tag{32}$$

$$\omega_1 = \frac{1}{\sqrt{3.116 \times 10^{-5}}} = 179.144\ \text{rad/s}, \quad f = 28.512\ \text{Hz}, \quad n = 1710.72\ \text{rpm}. \tag{33}$$

RESULTS AND DISCUSSION

The simulation in SolidWorks was performed with the parameters indicated in Table 1 to compare its results with the analytical calculations.

Geometric Modeling

A CAD model of the shaft was created as a solid cylindrical bar with a diameter of 15.875 mm and the length corresponding to each case (0.8 m for the overhang, 0.6 m for the simply supported ones). Disks were not modeled; remote masses were used.

Material and Boundary Conditions

AISI 4140 steel was assigned with E = 2.05 × 10¹¹ Pa, density 7850 kg/m³, and Poisson's ratio 0.3. The applied fixtures were:

For the simply supported cases: restriction of radial and axial translation at one end, only radial at the other (free rotation).

For the overhang: simple supports at x = 0 and x = 0.6 m, free end.

Application of Masses

Masses were introduced via Remote Mass by selecting the shaft face at the desired position, with a point mass of 2.5 kg, center of mass on the axis, and null moments of inertia.

Configuration and Execution of the Frequency Study

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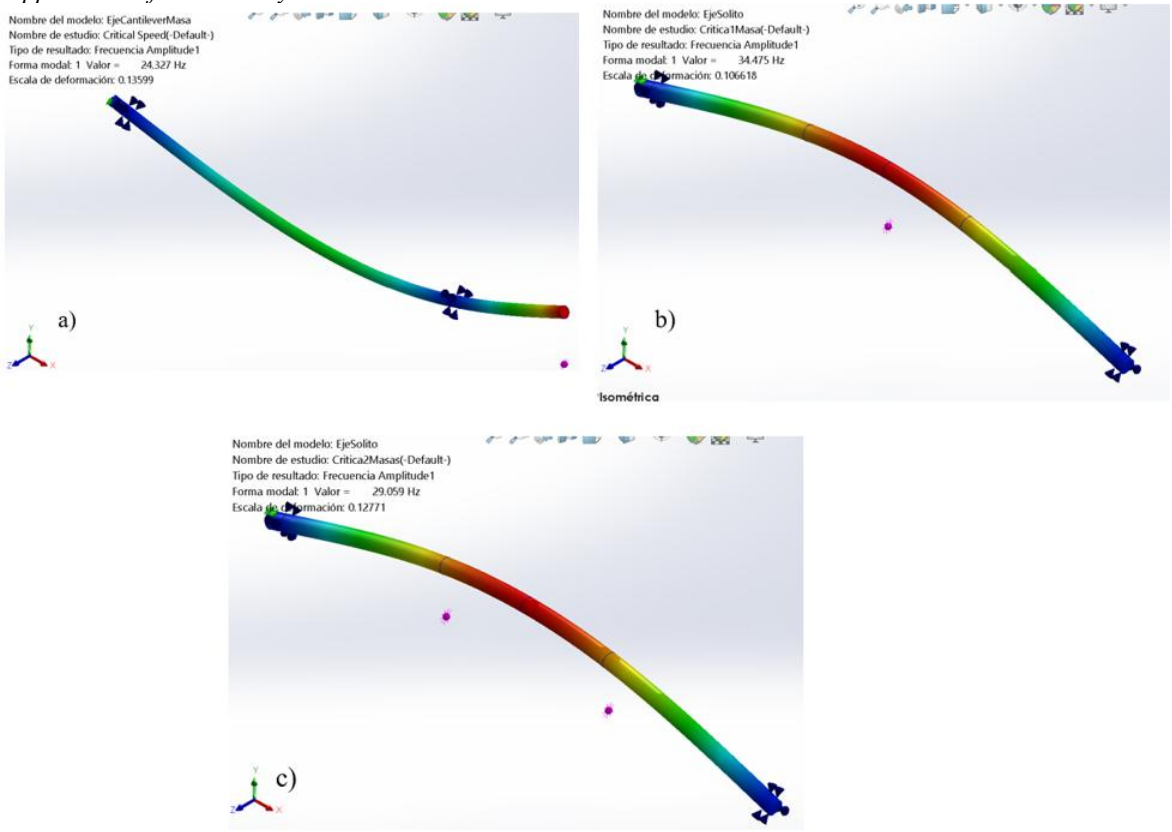
A Frequency study was created with 5 modes to calculate, Direct Sparse solver, without external loads. It was meshed with fine-sized tetrahedral elements (~5 mm) and the analysis was executed [6], [7].

Interpretation of Results

The frequency table and mode animation were reviewed. The first mode with predominant lateral bending deformation was identified with the support of the mass participation factor. Its frequency was used to estimate the first critical speed of the system. Figure 2a shows the mode corresponding to the overhang case.

Figure 2.

Results of the first critical speed: a) overhung shaft; b) simply supported shaft with central mass; c) simply supported shaft with two symmetric masses



Results and Interpretation

Tables 2, 3, and 4 present the calculated frequencies for each configuration. In cases with an overhang and two masses, the Dunkerley value is lower than the FEM result; in the central mass case, it is higher. The Rayleigh estimate included for the two-mass case exceeds the numerical result. These differences must be interpreted considering the mass and stiffness represented in each model, in addition to their support conditions.

Table 2.

Case 1: overhung shaft and mass at the end

Method	Frequency (Hz)	RPM
Shaft only (FEM)	83.903	5034
Disk only (analytical)	24.637	1478
Dunkerley combined	23.640	1418
FEM assembly (SW)	24.328	1459

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Table 3.
Case 2: simply supported shaft with central mass

Method	Frequency (Hz)	RPM
Shaft only (FEM)	88.530	5312
Disk only (analytical)	37.943	2277
Dunkerley combined	34.870	2093
FEM assembly (SW)	34.475	2069

Table 4.
Case 3: simply supported shaft with two symmetric centered masses

Method	Frequency (Hz)	RPM
Shaft only (FEM)	87.059	5223
Disk only (analytical)	31.17	1870
Dunkerley combined	28.512	1710.72
FEM assembly (SW)	29.06	1744

Table 5 compares the analytic estimates with FEM using their relative percentage difference.

Table 5.
Summary of relative error considering SolidWorks simulations

Case	Comparison	Error (%)
Overhang	Dunkerley vs. FEM	2.81
Central	Disk only anal. vs. FEM	9.1
Two masses	Rayleigh vs. FEM	6.7
Two masses	Dunkerley vs. FEM	1.7

Discussion

In all three configurations, Dunkerley's estimates are close to the FEM values presented in the tables. In the cases with an overhang and two masses, the Dunkerley calculation incorporates the frequency of the shaft without disks obtained by simulation. Rayleigh presents a larger discrepancy in the two-mass case, but this single case does not allow establishing a general relationship between the number of masses and the error.

Support conditions can modify the calculated frequencies and must be represented consistently when comparing models [5], [8], [9]. In the overhung case, FEM provides a higher value than Dunkerley. The presented comparison does not allow isolation of the effect of the fixtures on this difference.

As a didactic activity, the study allows proposing the following tasks:

Apply influence coefficients, static deflections, natural frequencies, and Rayleigh and Dunkerley methods to the analysis of the three models.

Contrast calculations with simulations and examine the hypotheses used in each procedure [6].

Model the shaft, define boundary conditions, and interpret vibration modes in SolidWorks.

These activities connect the formulation of the models with the interpretation of their results and allow discussing how mass, stiffness, and supports affect frequencies.

In preliminary design, analytical calculation can guide the selection of configurations that are later studied using more detailed numerical models [1], [3], [10]. Support stiffness and gyroscopic effects require treatment consistent with operating conditions [5], [9]. Extending the comparison to stepped shafts, non-symmetric masses, or elastic supports requires reviewing the hypotheses and the formulation of both models.

CONCLUSION

The study presents a comparison between analytical calculations and finite element simulation for three shaft configurations with concentrated masses. Dunkerley provides values close to the numerical frequencies indicated in the tables; Rayleigh shows a larger discrepancy in the two-mass case. The interpretation of these differences requires that both procedures represent the same geometry, mass, stiffness, and support conditions.

The calculations and models presented can be used to design activities that connect vibration theory with simulation. The study does not include a learning evaluation, so this application is proposed as a didactic possibility.

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The scope is limited to the three configurations and the hypotheses of the described models. The extension to stepped shafts, elastic supports, or systems where damping and gyroscopic effects are relevant requires specific analysis. A subsequent study could incorporate these conditions and compare calculated frequencies with experimental measurements on a test bench.

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STATEMENT ON THE USE OF ARTIFICAL INTELLIGENCE

AI was used as support for translation and assistance in bibliographic search without substituting the critical or methodological analysis in the preparation of this article and in the results obtained. The simulations are the product of tests conducted with engineering software.

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